



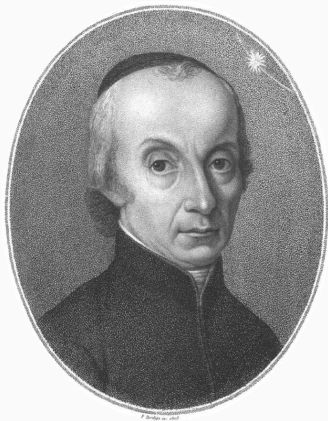
Machine Learning and the Physical World

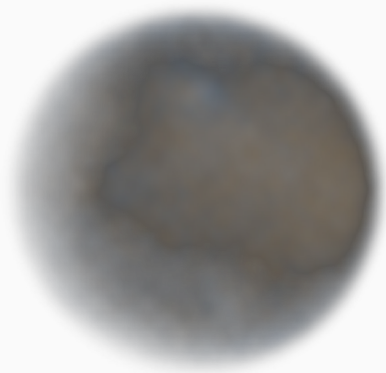
Lecture 1 : Introduction

Carl Henrik Ek - che29@cam.ac.uk

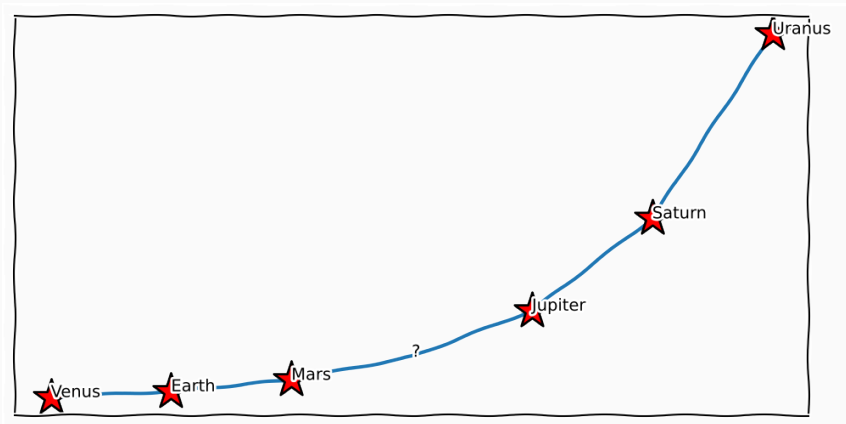
12th October, 2026

<http://carlhenrik.com>









$$a = 0.5 + 0.3 \times 2^m$$

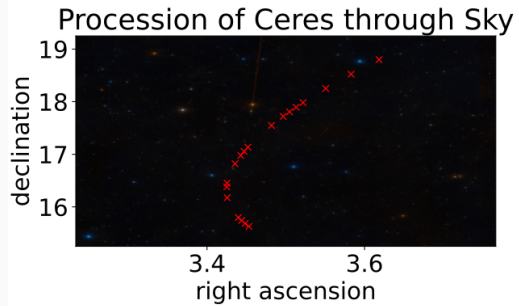
$$m = \{-\infty, 0, 1, 2, 3, 4, 5, 6, \dots\}$$

Beobachtungen des zu Palermo d. 1. Jan. 1801 von Prof. Piazzi neu entdeckten Gasteigns.

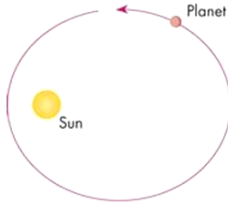
1801	Mittlere Sonnen- Zeit	Gerade Aufstieg in Zeit	Gerade Auf- steigung in Graden	Nördl. Abweich.	Geocentri- sche Länge	Geocentri- Breite	Ort der Sonne + 20" Aberration	Logar. d. Distanz ☉ ☿
	St. Min. Sec.	St. Min. Sec.	St. Min. Sec.	St. Min. Sec.	Z. Gr. Min. Sec.	St. Min. Sec.	Z. Gr. Min. Sec.	
Jan.	1 8 43 37.8	3 27 11.25	51 47 48.8	15 37 43.5	1 23 22 58.3	3 6 42.1	9 11 1 30.9	9.9926156
	2 8 39 4.6	3 26 53.85	51 43 27.8	15 41 5.5	1 23 19 44.3	3 2 24.9	9 12 2 28.6	9.9926317
	3 8 34 53.3	3 26 38.4	51 39 36.0	15 44 31.6	1 23 16 58.6	2 58 9.9	9 13 3 16.6	9.9926324
	4 8 30 42.1	3 26 23.15	51 35 47.3	15 47 57.6	1 23 14 15.5	2 53 55.6	9 14 4 14.9	9.9926418
	10 8 6 15.8	3 25 32.1	51 23 1.5	16 10 32.0	1 23 7 59.1	2 29 0.6	9 20 10 17.5	9.9927641
	11 8 2 17.5	3 25 29.73	51 22 26.6					
	13 7 54 26.2	3 25 30.30	51 22 34.5	16 22 49.5	1 23 10 27.6	2 16 59.7	9 23 12 13.8	9.9928490
	14 7 50 31.7	3 25 31.72	51 22 55.8	16 27 5.7	1 23 12 1.2	2 12 56.7	9 24 14 13.5	9.9928809
	17			16 40 13.0				
	18 7 35 11.3	3 25 55.1	51 28 45.0					
	19 7 31 28.5	3 26 8.15	51 32 2/3	16 49 16.1	1 23 25 59.2	1 53 38.2	9 29 19 53.8	9.9930607
	21 7 24 2.7	3 26 34.27	51 38 34.1	16 58 35.9	1 23 34 21.3	1 46 6.0	10 1 20 40.3	9.9931434
	22 7 20 21.7	3 26 49.42	51 42 21.3	17 3 18.5	1 23 39 1.8	1 42 28.1	10 2 21 32.0	9.9931886
	23 7 16 45.5	3 27 6.90	51 46 43.5	17 8 5.5	1 23 44 15.7	1 38 52.1	10 3 22 22.7	9.9932348
	28 6 58 51.3	3 28 54.53	52 13 38.3	17 32 54.1	1 24 15 15.7	1 21 6.9	10 8 26 20.1	9.9935061
	30 6 51 52.9	3 29 48.14	52 27 2.1	17 43 11.0	1 24 30 9.0	1 14 16.0	10 10 27 46.2	9.9936332
	31 6 48 26.4	3 30 17.25	52 34 18.8	17 48 21.5	1 24 38 7.3	1 10 54.6	10 11 28 28.5	9.9937007
Febr.	1 6 44 59.9	3 30 47.2	52 41 48.0	17 53 36.3	1 24 46 19.3	1 7 30.9	10 12 29 9.6	9.9937703
	2 6 41 35.8	3 31 19.06	52 49 45.9	17 58 57.5	1 24 54 57.9	1 4 1.5	10 13 29 49.9	9.9938423
	5 6 31 31.5	3 33 2.70	53 15 40.5	18 15 1.0	1 25 22 43.4	0 54 23.9	10 16 31 45.5	9.9940751
	8 6 21 39.2	3 34 58.50	53 44 37.5	18 31 23.2	1 25 53 29.5	0 45 5.0	10 19 33 33.3	9.9943276
	11 6 11 58.2	3 37 6.54	54 16 38.1	18 47 58.8	1 26 26 40.0	0 36 2.9	10 22 35 11.4	9.9945823

A NEW PLANET.

An important circumstance in Astronomy has just occurred, no less than the Discovery of ANOTHER NEW PLANET!!! This celestial phenomenon moves between the orbits of Mars and Jupiter, and is an intermediate Planet between them. It was discovered by M. PIAZZI, an Italian Astronomer, on the 1st of January, 1801. He concealed the discovery, to preserve all the honour and observations to himself, till after six weeks close watching, he fell ill. It will not be in a situation, with regard to the Sun, to be observed again, till a month or two hence. It is but a small Planet, ranking only as a Star of the eighth magnitude, and therefore not visible to the naked eye. Its motion is nearly parallel to the ecliptic, at present about $4\frac{1}{2}^{\circ}$ to the north of it, and nearly entering the sign Leo. The distance from the Sun is about $2\frac{1}{2}$ times that of the earth, and the periodical time nearly four years and two months.—Other particulars shall be given in our next.

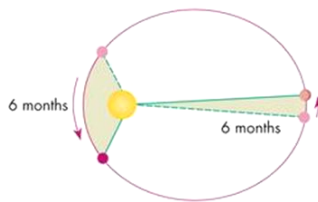


Kepler's 3 Laws of Planetary Motion



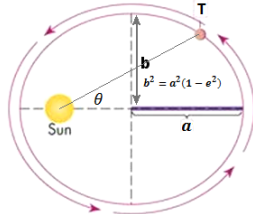
(1)

The orbits are ellipses



(2)

Equal areas in equal time



(3)

T = time to complete orbit
 $T^2 \propto a^3$ a = semi-major axis



hier in der Nähe der Quadratur der Einfluss der Sonnen-Länge geringer ist, als in andern Lagen. Dr. *Gauß* glaubt daher, daß es nicht unendlich wäre, wenn man die Fehler der Sonnentafeln aus sehr genauen Beobachtungen für diese Zeiten bestimmte, und die Örter der Sonne hiernach verbesserte. Diese vier-
ten Elemente sind nun folgende:

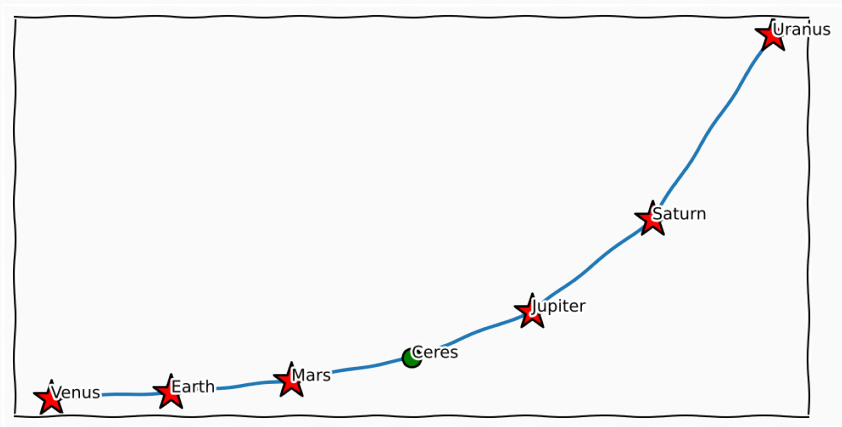
Sonnenferne	326° 27' 38"	Hieraus:	
Ö	51 0 44	größte Mittelp. Glief	
Neigung	10 36 57	chung	6° 27' 41"
Log. halb. gr. Axe	0,4420527	Log. mittlere helioc.	
Excentricität	0,0825087	tropische Beweg.	370° 914
Epoch 1800 31 Dec. 77	26' 34"		

Aus diesen Elementen hat Dr. *Gauß* folgende Örter der *Ceres Ferdinandea* im voraus berechnet. Die Zeit ist mittlere für Mitternacht in *Palermo*.

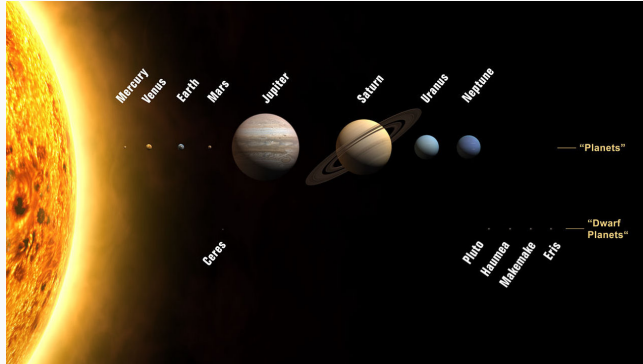
1801	Geocen- trische Länge	Geo- centr. Ab- stand nach	Logarith. des Ab- standes von der S	Logarith. des Ab- standes von der G	Verhält- niß der auf- gehenden Heliocl.
	Z				
Nov. 25	5 20 16	0 25	0,42181	0,40468	0,6102
Dec. 1	5 22 15	0 48	0,40940	0,40472	0,6459
	7 5 24	7 10 12	0,39644	0,40470	0,6855
	13 5 25	10 37	0,38296	0,40488	0,7290
	19 5 27	11 4	0,36902	0,40499	0,7770
	25 5 28	11 32	0,35468	0,40512	0,8295
	31 6 0	10 12	0,34000	0,40520	0,8869

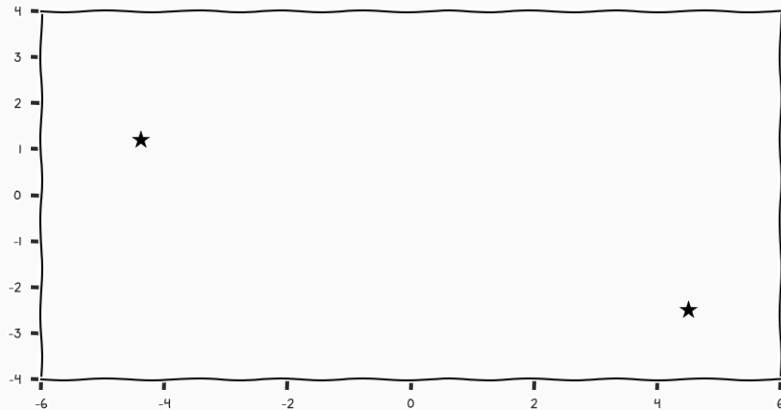
Sollte man den Ort des Planeten nach diesen Elementen genauer, oder auf eine längere Zeit berechnen wollen: so setzen wir zu diesem Behufe noch folgende Formeln hierher:

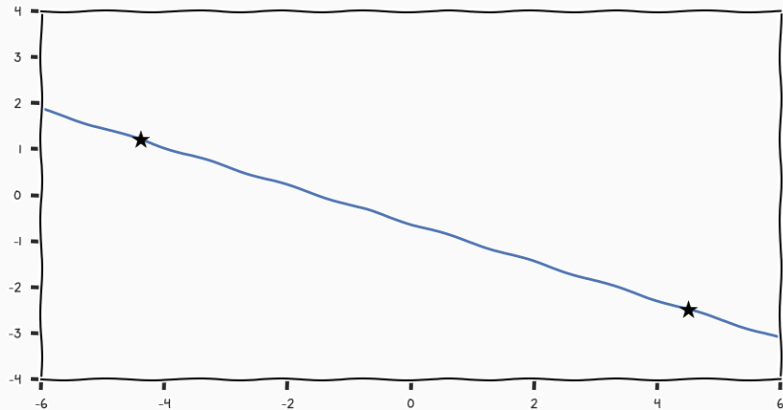
1) Zur



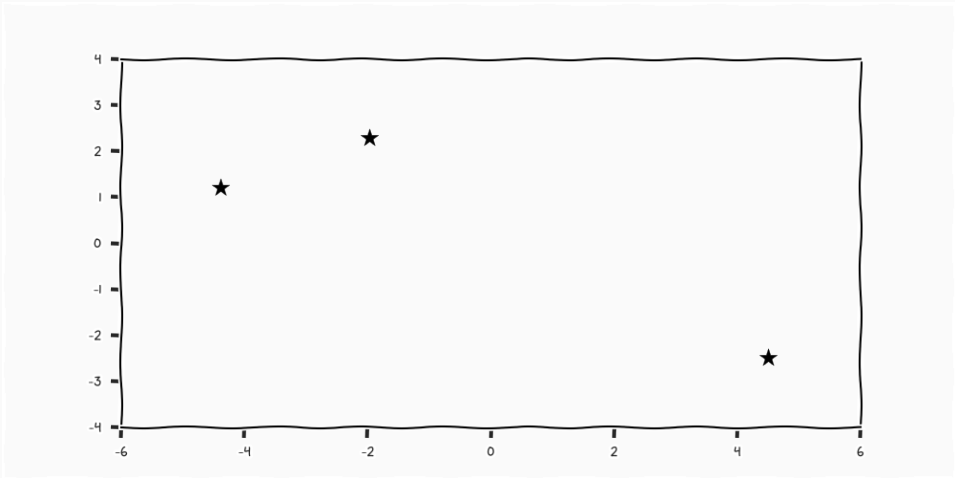




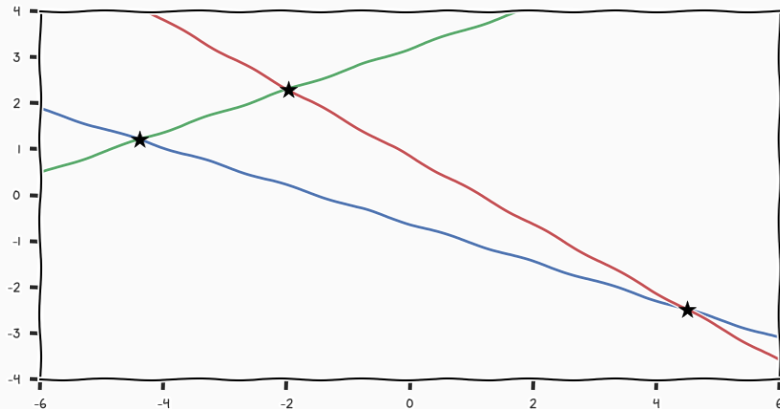




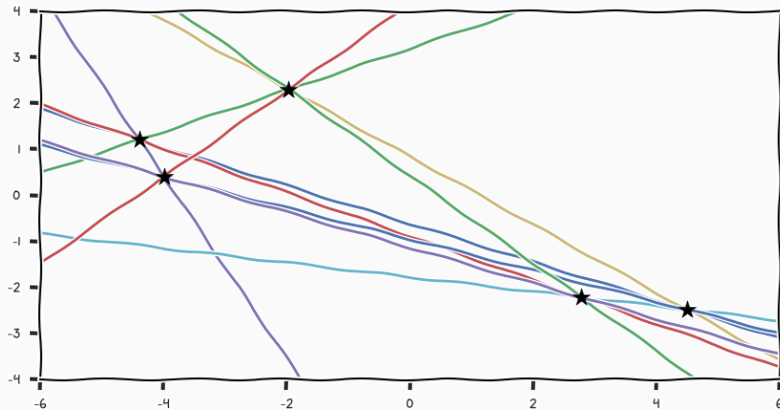
An Over-determined System

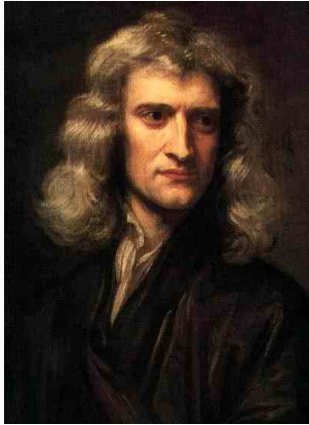


An Over-determined System



An Over-determined System





An Over-determined System

- An over-determined system means that we have more data than we need to determine our parameters.

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- The problem arises because the **model** is a simplification of the **world** and the data is therefore **inconsistent** with our model.



We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

Data + Model $\rightarrow$ Prediction

This led to the idea of scientific determinism, which seems first to have been publicly expressed by the French scientist, Laplace.
– Stephen Hawkins

¹Does God Play Dice? - Stephen Hawkins

All these efforts in the search for truth tend to lead [the human mind] back continually to the vast intelligence which we have just mentioned, but from which it will always remain infinitely removed.

– Pierre Simon Laplace, *A Philosophical Essay on Probabilities*
Laplace, 1814

"The curve described by a simple molecule of air or vapor is regulated in a manner just as certain as the planetary orbits; the only difference between them is that which comes from our ignorance. Probability is relative, in part to this ignorance, in part to our knowledge. We know that of three or greater number of events a single one ought to occur; but nothing induces us to believe that one of them will occur rather than the others. In this state of indecision it is impossible for us to announce their occurrence with certainty. It is, however, probable that one of these events, chosen at will, will not occur because we see several cases equally possible which exclude its occurrence, while only a single one favors it."

$$p(\text{Data}) + p(\text{Model}) \rightarrow p(\text{Prediction})$$

- Over determined system

$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

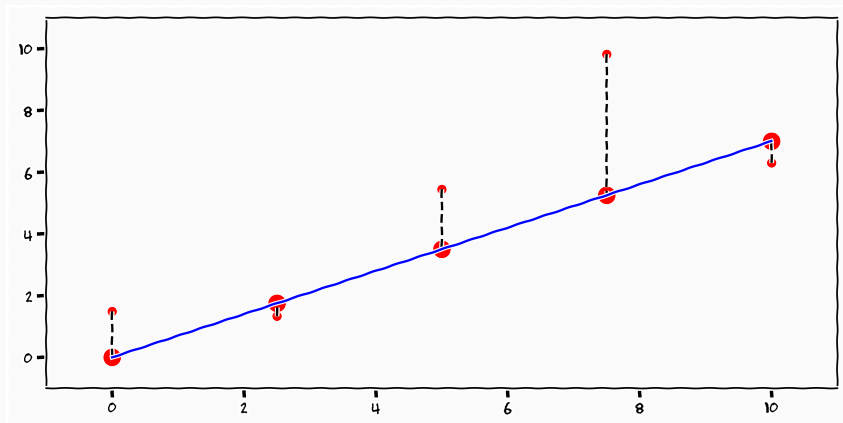
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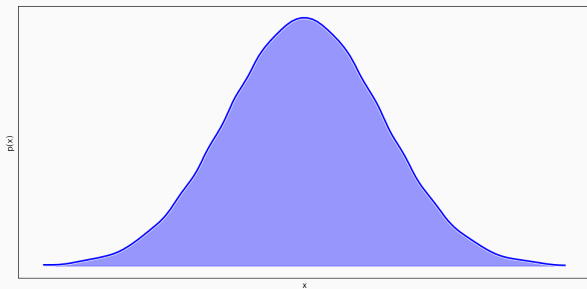
- Parametrisation of ignorance

$$y_i = [a, b] \begin{bmatrix} x_i \\ 1 \end{bmatrix} + \epsilon_i$$

Decomposition



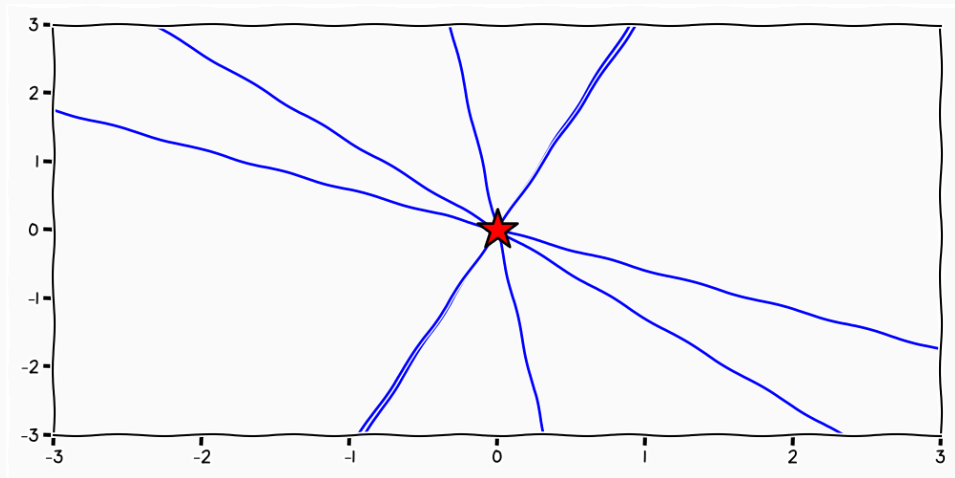
The Gaussian Distribution

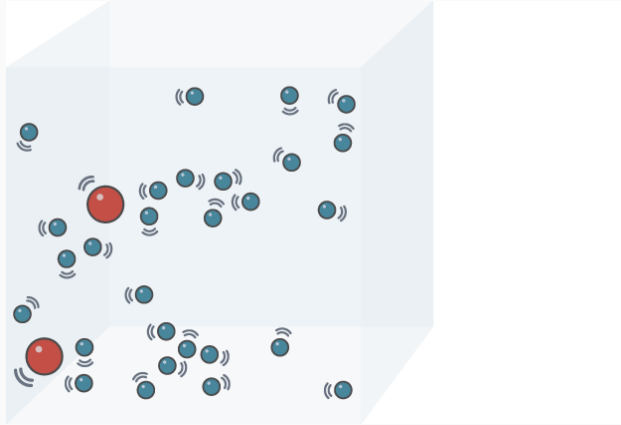


$$y_i = [a, b] \begin{bmatrix} x_i \\ 1 \end{bmatrix} + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \cdot)$$

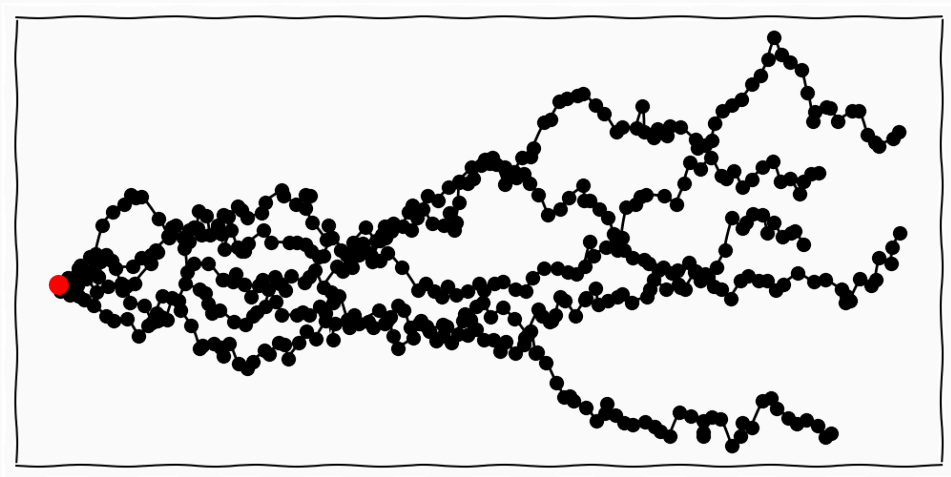
$$p(y_i \mid \mathbf{x}_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_i - \mathbf{w}^T \mathbf{x}_i)^2}{2\sigma^2}\right)$$

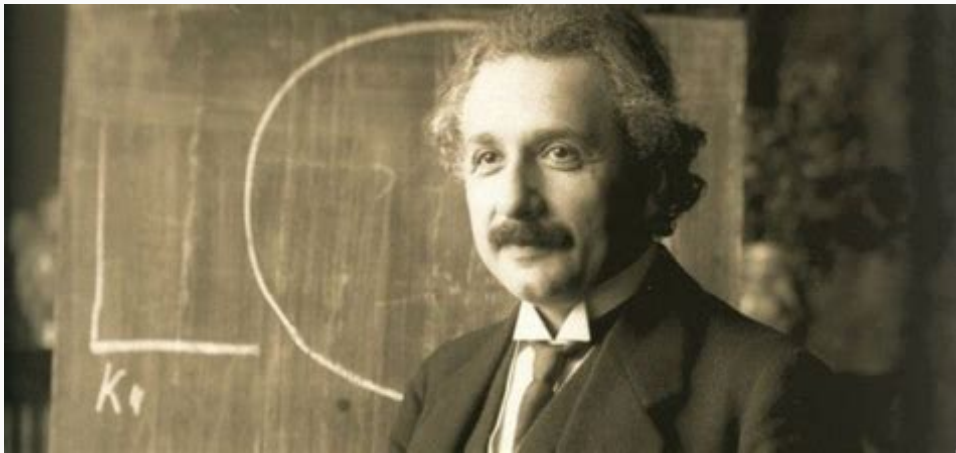
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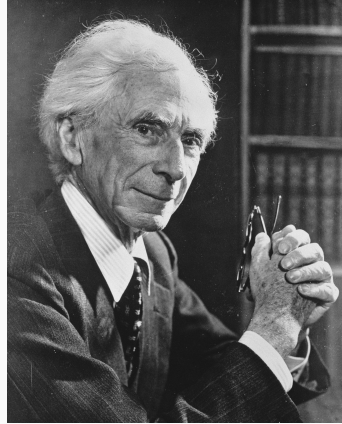
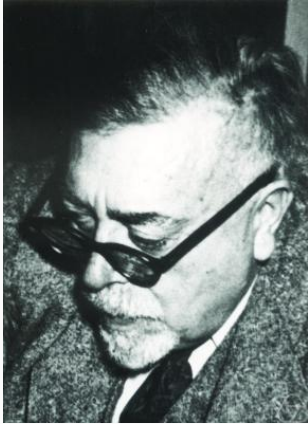












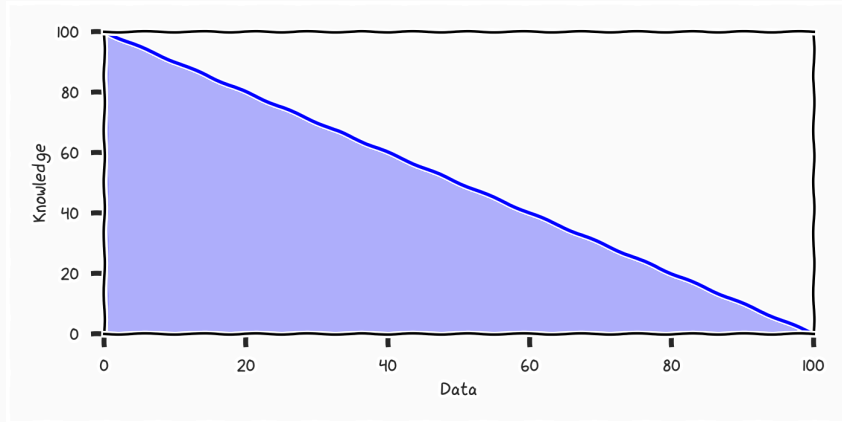
Machine Learning in the Physical World

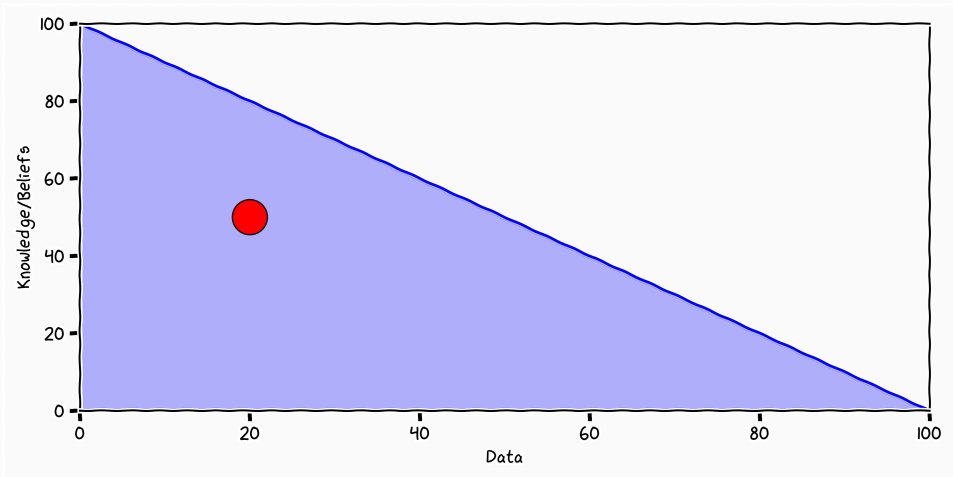


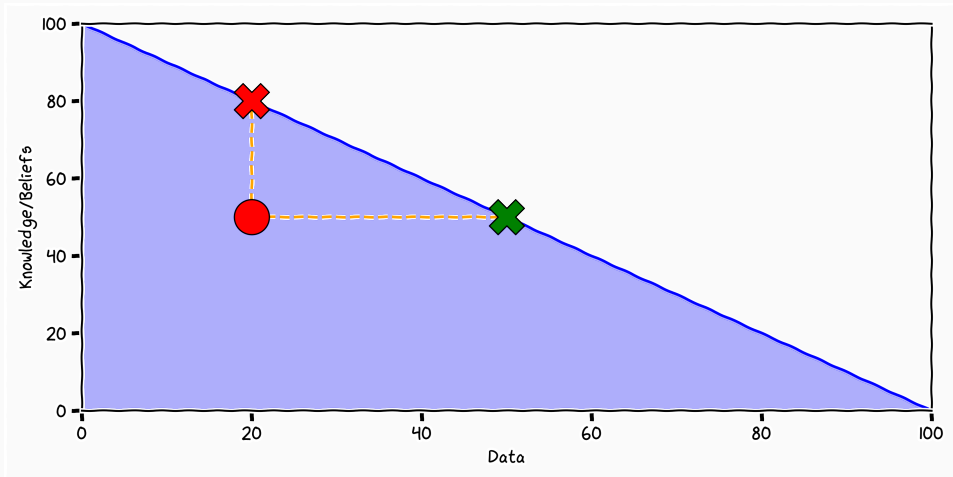
Data + Model $\xrightarrow{\text{Compute}}$ Prediction

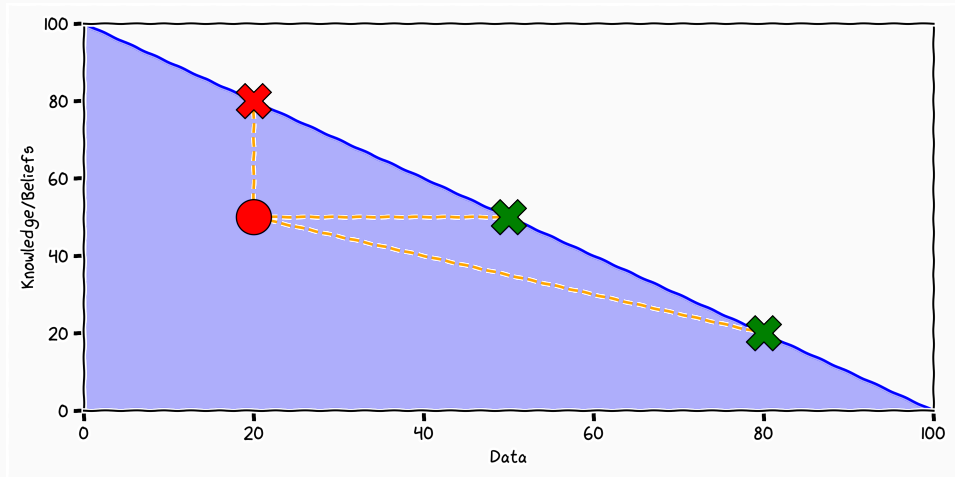
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Compute

Data + Model $\rightarrow$ Prediction

What is this course not

- Not about models

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- Not about inference

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- Not about models
- Not about inference
- Not about specific problems

Week 1 Introduction

Week 1 Simulation

Week 2 Quantification of Beliefs

Week 2 Gaussian Processes

Week 3 Practical Gaussian
Processes

Week 3 Emulation

Week 4 Sequential Decision
Making Under
Uncertainty

Week 4 Reinforcement Learning

Week 5 Probabilistic Numeric

Week 5 Experimental Design

Week 6 Sensitivity Analysis

Week 6 Multi-fidelity Modelling

- Individual Assessment (2 · 15%)
 - Gaussian Processes **deadline** 30/10
 - Sequential Decision Making **deadline** 13/11

- Group Project (70%)
 - pick your own simulation environment
 - **deadline** 20/1

Week 7 Project Guest Lecture

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Week 7 Generative Models

Week 7 Project Guest Lecture

Week 7 Generative Models

Week 8 Research Talk

Week 7 Project Guest Lecture

Week 7 Generative Models

Week 8 Research Talk

Week 8 TBD

<http://carlhenrik.com/l48-mlpw>

- Lecture Notes
- Worksheets



Summary

- The history of making a problem well posed by mixing data and knowledge goes far back

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 - model
 - data
 - compute

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